This set contains three pages (beginning with this page)
All questions must be answered
Questions 1 and 2 each weigh 25 % while question 3 weighs 50 %. These weights, however, are only indicative for the overall evaluation.

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MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO JUNE 24 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the Calvo pricing model, expectations about future marginal costs are irrelevant for firms that by chance are able to reset their prices.

A False. Future marginal costs are relevant, as a firm that sets its price in a given period, knows that with some probability it will be "stuck" with that price in the next period, and the next, and so on. Hence, expectations about future economic conditions summarized by marginal costs are relevant for a price-changing firm.

(ii) In a flex-price, cash-in-advance model, it is never possible to determine the optimal inflation rate.

A False. Only in special case where the inflation rate and the nominal interest rate do not distort any economic decisions, will different inflation rates be irrelevant. This could be the case where only consumption gives utility. But as long as the cash-in-advance constraint binds, inflation and the nominal interest rate may affect labor supply, capital accumulation or other decisions leading to a well-defined optimal monetary policy. Often this will be associated with a zero nominal interest rate, as this eliminates the distortion of the constraint (i.e., implementation of the Friedman rule).

(iii) In rational-expectations settings where real effects of monetary policy are due to unexpected policy movements, systematic components of monetary policy have no real effects.

A False. This so-called Policy Irrelevance Proposition rarely holds in theory (or in practice). Many examples can be given. For example, in the Barro and Gordon model, systematic shifts in central bank preferences result in different output/inflation variabilities. Also, in the standard New-Keynesian model, different policy-rule parameters will imply different propagation of shocks.

QUESTION 2:

Money in the utility function: Finding money demand
Consider an infinite-horizon economy in discrete time, where utility of the representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t u\left(c_t, m_t\right), \qquad 0 < \beta < 1, \tag{1}$$

with

$$u\left(c_t, m_t\right) \equiv b \ln c_t + (1 - b)\left(m^F \ln m_t - m_t\right), \qquad 0 < b < 1, \quad m^F > 0.$$

Agents maximize utility subject to the budget constraint

$$\begin{aligned} c_t + k_t + m_t &= f\left(k_{t-1}\right) + \tau_t + (1 - \delta)\, k_{t-1} + \frac{1}{1 + \pi_t} m_{t-1}, \qquad 0 < \delta < 1 \\ &\equiv \omega_t, \end{aligned} \tag{2}$$

where c_t is consumption, m_t is real money balances at the end of period t, k_{t-1} is physical capital, τ_t are monetary transfers from the government, and π_t is the inflation rate. Function f satisfies $f' > 0$, $f'' < 0$.

(i) Derive the relevant first-order conditions for optimal behavior [Hint: Set up the value function $V\left(\omega_t\right) = \max_{c_t, m_t} \left\{u\left(c_t, m_t\right) + \beta V\left(\omega_{t+1}\right)\right\}$ and substitute out ω_{t+1} by (2) and k_t by $k_t = \omega_t - c_t - m_t$]

A Using the hint, one recovers the following first-order conditions:

$$\begin{aligned} u_c(c_t, m_t) - \beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta\right] &= 0, \\ u_m(c_t, m_t) - \beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta - \frac{1}{1+\pi_{t+1}}\right] &= 0. \end{aligned}$$

(ii) Interpret the first-order conditions intuitively, and show that they can be combined into

$$\frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} = \frac{i_t}{1+i_t}, \tag{3}$$

where $f_k(k_t) + 1 - \delta = (1+i_t)/(1+\pi_{t+1})$ defines i_t as the nominal interest rate. Discuss this and discuss whether steady-state superneutrality holds in the model.

A The first condition states consumption is chosen such that its marginal benefit (in terms of its marginal utility) equals its marginal cost (in terms of the discounted marginal value cost of lower next-period wealth times the real interest rate). The second condition states real money balances are chosen such that their marginal benefits (in terms of marginal utility and discounted marginal value gain of higher next-period wealth corrected by inflation) equal their marginal cost (in terms of the discounted marginal value cost of lower next-period wealth times the real interest rate). Combining the two conditions gives

$$\begin{aligned} \frac{u_m(c_t, m_t)}{u_c(c_t, m_t)} &= \frac{\beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta - \frac{1}{1+\pi_{t+1}}\right]}{\beta V'(\omega_{t+1})\left[f_k(k_t) + 1 - \delta\right]} \\ &= 1 - \frac{1}{\left[f_k(k_t) + 1 - \delta\right](1+\pi_{t+1})} \\ &= 1 - \frac{1}{1+i_t} \\ &= \frac{i_t}{1+i_t}, \end{aligned}$$

where the next-to-last line uses the definition of the nominal interest rate. In this model, there will be steady-state superneutrality, as any change in the nominal interest rate only affects money holdings for given consumption. Real money holdings do not affect the marginal utility of consumption (or the marginal product of capital), so the model's steady state for capital is given by $1/\beta = f_k(k^{ss}) + 1 - \delta$ which is independent of monetary factors (the determination of the steady state capital stock can be shown formally by using the result from the Envelope Theorem, $V'(\omega_t) = \beta V'(\omega_{t+1})[f_k(k_t) + 1 - \delta]$, but a verbal account is sufficient).

(iii) Apply the particular functional form of u and characterize the monetary policy that maximizes the utility of the representative agent and find the corresponding optimal steady-state real balances. Explain the results intuitively.

A With the particular utility function, equation (3) gives the following steady-state relationship:

$$\frac{(1-b)\left(m^F/m^{ss}-1\right)}{b/c^{ss}} = \frac{i^{ss}}{1+i^{ss}}.$$

As monetary policy "only" affects m^{ss}, the welfare-maximizing policy is one that induces $u_m(c_t, m_t) = 0$, which is equivalent to $i^{ss} = 0$. I.e., the Friedman rule where the opportunity cost of money balances is zero. In this case, it implies $m^{ss} = m^F$, where m^F is the optimum quantity of money in the model.

QUESTION 3:

Monetary policy trade offs

Consider the following log-linear model of a closed economy:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}(i_t - \mathrm{E}_t\pi_{t+1} - r_t^n), \qquad \sigma > 0, \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

where x_t is the output gap, i_t is the nominal interest rate (the monetary policy instrument), π_t is goods price inflation and r_t^n is the natural rate of interest, which is assumed to be a mean-zero, serially uncorrelated shock. E_t is the rational expectations operator conditional on all information up to and including period t.

(i) Discuss the economic mechanisms behind equations (1) and (2).

A Here is should be mentioned that (1), a dynamic IS curve, is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, i_t- $\mathrm{E}_t\pi_{t+1}$, make consumers increase future consumption relative to current. Equation (2), a New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. The more price rigidity (e.g., the lower a probability of price adjustment under a Calvo price-setting scheme), the smaller is κ. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be effective for some periods.

(ii) Assume that the monetary authority wants to minimize the loss function

$$L = \frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda x_t^2 + \pi_t^2\right], \qquad \lambda > 0. \tag{3}$$

Discuss the economic foundations for this loss function.

A This type of loss function can be derived as the second-order Taylor approximation to (the negative of) the representative household's utility function. Price rigidity causes losses from aggregate mark-ups being different from the desired markup, and under the Calvo-price structure, staggering cause inefficient dispersion of consumption of various goods. The quadratic terms in (3) reflect the costs from these fluctuations. Inflation is proportional to the inefficient goods dispersion, and output gap fluctuations are proportional to the fluctuations in the mark-up gap (that causes inefficient fluctuations in consumption and labor).

(iii) Derive the optimal values of x_t and π_t under discretionary policymaking [Hint: Consider x_t the policy instrument, and acknowledge that under discretion the optimization problem becomes a sequence of static problems as expected values can be taken as given]. Discuss the solutions, and describe how the nominal interest rate will move with the natural rate of interest.

A Using the hint, the first-order condition is found as

$$\lambda x_t + \kappa \pi_t = 0.$$

This is inserted into (2) in order to substitute out x_t:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} - \left(\kappa^2/\lambda\right) \pi_t$$

which yields

$$\pi_t = \left[\beta/\left(1 + \kappa^2/\lambda\right)\right] \mathrm{E}_t \pi_{t+1}.$$

Since the characteristic root of this difference equation is unstable $((1+\kappa^2/\lambda)/\beta > 1)$, the unique non explosive solution for inflation is $\pi_t = 0$ all t. In combination with the first-order condition, one recovers $x_t = 0$ as well. Hence, under discretionary policymaking, inflation and the output gap is fully stabilized (leading to the lowest possible welfare loss). In terms of the actual policy instrument, the nominal interest rate, it follows from (1) that it will be given by $i_t = r_t^n$. Hence, the policy rate is constantly adjusted so as to follow the natural rate of interest. In consequence, aggregate demand is adjusted to equal aggregate supply under flexible prices at all times. Hence, output follows the natural rate of output leaving the output *gap* unchanged.

(iv) Assume that the monetary policymaker instead follows a rule for nominal interest-rate setting given as

$$i_t = \phi \pi_t, \qquad \phi > 1. \tag{4}$$

Derive the solutions for x_t and π_t for the system (1), (2) and (4). [Hint: Conjecture that the solutions are linear functions of the period's natural rate of interest, r_t^n, and remember that $\mathrm{E}_t r_{t+1}^n = 0$.]. Discuss the differences between these solutions and the ones obtained under discretionary policymaking. Can the monetary policy rule (4) be parameterized such that it will "deliver" the outcomes under discretionary policymaking?

A Use the hint and make the following conjectures:

$$\begin{aligned} x_t &= X r_t^n, \\ \pi_t &= Y r_t^n, \end{aligned}$$

where X and Y are the unknown parameters to be determined. Note that the assumptions about the stochastic nature of the shock imply that $\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0$. Inserting the conjectures, along with (4), into (1) and (2) therefore give:

$$\begin{aligned} X r_t^n &= -\sigma^{-1} \left(\phi Y r_t^n - r_t^n \right) \\ Y r_t^n &= \kappa X r_t^n. \end{aligned}$$

This verifies the form of the conjecture (and since $\phi > 1$ we know the solution is unique—a good thing to mention, although not required here), and identifies the parameters by

$$\begin{aligned} X &= -\sigma^{-1}(\phi Y - 1) \\ Y &= \kappa X \end{aligned}$$

and thus

$$\begin{aligned} X\left(1 + \sigma^{-1}\phi\kappa\right) &= \sigma^{-1} \\ X &= \frac{\sigma^{-1}}{1 + \sigma^{-1}\phi\kappa} \end{aligned}$$

and

$$Y = \frac{\sigma^{-1}\kappa}{1 + \sigma^{-1}\phi\kappa}$$

The solutions for the output gap and inflation are therefore given as

$$\begin{aligned} x_t &= \frac{\sigma^{-1}}{1 + \sigma^{-1}\phi\kappa} r_t^n, \\ \pi_t &= \frac{\sigma^{-1}\kappa}{1 + \sigma^{-1}\phi\kappa} r_t^n. \end{aligned}$$

In contrast with discretionary policy, the rule implies fluctuations in output gap and inflation when there are fluctuations in the natural rate of interest. With the policy rule, a change in the natural rate of interest is not fully met by a change in the policy rate. Hence, a positive value of r_t^n increases the output gap and inflation. The real interest rate (which equals the nominal in this simple model) only increases by $\phi\pi_t = \frac{\sigma^{-1}\phi\kappa}{1+\sigma^{-1}\phi\kappa} r_t^n < r_t^n$. The more aggressive is the response to inflation changes, a higher ϕ, the higher is the interest rate change, and the more stable will inflation and output gap become. In the limit of $\phi \to \infty$ the solutions approach the optimal one involving full stabilization of both inflation and the output gap. This is possible as the shock to the natural rate of interest, does not pose a trade off for the monetary policymaker.

(v) Will an ability to conduct optimal policy under commitment be advantageous in this setting? Why/Why not?

A Under commitment, the policymaker can affect expectations. This is an ability that can never make the policymaker worse of (among the particular ways of affecting expectations are the one that involves no changes in expectations). Hence, under commitment the policymaker can obtain full stabilization of the output gap and inflation just as under discretion. It cannot do better than that, so commitment does not provide a gain in this setting where full stabilization of both macro variables is possible. (It could be noted that if trade-offs are involved in policymaking, commitment usually is advantageous as the policymaker can improve trade-offs by affecting inflation expectations.)